

$$1. \quad \forall x \in \mathbb{R} : \cos^2 x + \sin^2 x = 1 \quad \forall x \in \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\} : 1 + \operatorname{tg}^2 x = \frac{1}{\cos^2 x}$$

$$\forall x \in \mathbb{R} \setminus \{k\pi, k \in \mathbb{Z}\} : 1 + \cot g^2 x = \frac{1}{\sin^2 x}.$$

- $\forall x \in \mathbb{R} : \cos(x + 2k\pi) = \cos x ; \forall x \in \mathbb{R} : \sin(x + 2k\pi) = \sin x$
- $\forall x \in \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi; k \in \mathbb{Z} \right\} : \operatorname{tg}(x + k'\pi) = \operatorname{tg} x; (k' \in \mathbb{Z})$
- $\forall x \in \mathbb{R} \setminus \{k\pi; k \in \mathbb{Z}\} : \cot g(x + k'\pi) = \cot g x; (k' \in \mathbb{Z})$
- $\forall x \in \mathbb{R} \setminus \left\{ \frac{k\pi}{2}, k \in \mathbb{Z} \right\} : \operatorname{tg} x \times \cot g x = 1$

## 2. Formules relatives aux angles associés :

|                                                                               |                                                                              |                                                                                                                                                     |                                                                                                                                                               |
|-------------------------------------------------------------------------------|------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\cos(-x) = \cos x \quad \forall x \in \mathbb{R}$                            | $\sin(-x) = -\sin x \quad \forall x \in \mathbb{R}$                          | $\operatorname{tg}(-x) = -\operatorname{tg} x$<br>$\forall x \in \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}$      | $\cot g(-x) = -\cot g x$<br>$\forall x \in \mathbb{R} \setminus \{k\pi, k \in \mathbb{Z}\}$                                                                   |
| $\cos(\pi - x) = -\cos x \quad \forall x \in \mathbb{R}$                      | $\sin(\pi - x) = \sin x \quad \forall x \in \mathbb{R}$                      | $\operatorname{tg}(\pi - x) = -\operatorname{tg} x$<br>$\forall x \in \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi; k \in \mathbb{Z} \right\}$ | $\cot g(\pi - x) = -\cot g x$<br>$\forall x \in \mathbb{R} \setminus \{k\pi, k \in \mathbb{Z}\}$                                                              |
| $\cos(\pi + x) = -\cos x \quad \forall x \in \mathbb{R}$                      | $\sin(\pi + x) = -\sin x \quad \forall x \in \mathbb{R}$                     | $\operatorname{tg}(\pi + x) = \operatorname{tg} x$<br>$\forall x \in \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi; k \in \mathbb{Z} \right\}$  | $\cot g(\pi + x) = \cot g x$<br>$\forall x \in \mathbb{R} \setminus \{k\pi, k \in \mathbb{Z}\}$                                                               |
| $\cos\left(\frac{\pi}{2} - x\right) = \sin x \quad \forall x \in \mathbb{R}$  | $\sin\left(\frac{\pi}{2} - x\right) = \cos x \quad \forall x \in \mathbb{R}$ | $\operatorname{tg}\left(\frac{\pi}{2} - x\right) = \cot g x$<br>$\forall x \in \mathbb{R} \setminus \{k\pi; k \in \mathbb{Z}\}$                     | $\cot g\left(\frac{\pi}{2} - x\right) = \operatorname{tg} x$<br>$\forall x \in \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}$  |
| $\cos\left(\frac{\pi}{2} + x\right) = -\sin x \quad \forall x \in \mathbb{R}$ | $\sin\left(\frac{\pi}{2} + x\right) = \cos x \quad \forall x \in \mathbb{R}$ | $\operatorname{tg}\left(\frac{\pi}{2} + x\right) = -\cot g x$<br>$\forall x \in \mathbb{R} \setminus \{k\pi; k \in \mathbb{Z}\}$                    | $\cot g\left(\frac{\pi}{2} + x\right) = -\operatorname{tg} x$<br>$\forall x \in \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}$ |

## 3. Formules d'addition :

- $\cos(a + b) = \cos a \times \cos b - \sin a \times \sin b \quad \forall a \in \mathbb{R} \quad \forall b \in \mathbb{R}$   
 $\cos(a - b) = \cos a \times \cos b + \sin a \times \sin b \quad \forall a \in \mathbb{R} \quad \forall b \in \mathbb{R}$
- $\sin(a + b) = \sin a \times \cos b + \cos a \times \sin b \quad \forall a \in \mathbb{R} \quad \forall b \in \mathbb{R}$   
 $\sin(a - b) = \sin a \times \cos b - \cos a \times \sin b \quad \forall a \in \mathbb{R} \quad \forall b \in \mathbb{R}$
- $\operatorname{tg}(a + b) = \frac{\operatorname{tga} + \operatorname{tgb}}{1 - \operatorname{tga} \times \operatorname{tgb}} \quad \forall a \in D, \forall b \in D, \forall (a + b) \in D : D = \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}$

- $\operatorname{tg}(a-b) = \frac{\operatorname{tga} - \operatorname{tgb}}{1 + \operatorname{tga} \times \operatorname{tgb}} \quad \forall a \in D, \forall b \in D, \forall (a-b) \in D : D = \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}$

4. Formules de multiplication par 2 :

- $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x \quad \forall x \in \mathbb{R}.$
- $\sin 2x = 2\sin x \times \cos x \quad \forall x \in \mathbb{R}$
- $\operatorname{tg} 2x = \frac{2\operatorname{tg} x}{1 - \operatorname{tg}^2 x} ; (x \in \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}; 2x \in \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\})$

5. Formules de transformation de sommes en produits :

- $\cos p + \cos q = 2\cos \frac{p+q}{2} \times \cos \frac{p-q}{2}, \forall p \in \mathbb{R}, \forall q \in \mathbb{R}$
- $\cos p - \cos q = -2\sin \frac{p+q}{2} \times \sin \frac{p-q}{2}, \forall p \in \mathbb{R}, \forall q \in \mathbb{R}$
- $\sin p + \sin q = 2\sin \frac{p+q}{2} \times \cos \frac{p-q}{2}, \forall p \in \mathbb{R}, \forall q \in \mathbb{R}$
- $\sin p - \sin q = 2\cos \frac{p+q}{2} \times \sin \frac{p-q}{2}, \forall p \in \mathbb{R}, \forall q \in \mathbb{R}$

6. Formules de transformation de produits en sommes :

- $\cos a \times \cos b = \frac{1}{2} [\cos(a+b) + \cos(a-b)]$
- $\sin a \times \sin b = -\frac{1}{2} [\cos(a+b) - \cos(a-b)]$
- $\sin a \times \cos b = \frac{1}{2} [\sin(a+b) + \sin(a-b)]$

7. transformation de  $a \times \cos x + b \times \sin x$  :

$$a \cos x + b \sin x = r \cos(x - \varphi) ; r = \sqrt{a^2 + b^2}; \cos \varphi = \frac{a}{r}; \sin \varphi = \frac{b}{r}; (a \in \mathbb{R}^*, b \in \mathbb{R}^*)$$

8. Si on pose :  $t = \operatorname{tg} \frac{x}{2}$  alors  $\cos x = \frac{1-t^2}{1+t^2}$  ;  $\sin x = \frac{2 \times t}{1+t^2}$  ;  $\operatorname{tg} x = \frac{2 \times t}{1-t^2}$

9. Equations trigonométriques :

- $\cos x = \cos \alpha \Leftrightarrow \begin{cases} x = \alpha + 2k\pi, k \in \mathbb{Z} \\ \text{ou} \\ x = -\alpha + 2k\pi, k \in \mathbb{Z} \end{cases}$
- $\sin x = \sin \alpha \Leftrightarrow \begin{cases} x = \alpha + 2k\pi, k \in \mathbb{Z} \\ \text{ou} \\ x = \pi - \alpha + 2k\pi, k \in \mathbb{Z} \end{cases}$
- $\operatorname{tg} x = \operatorname{tg} y \Leftrightarrow x = y + k\pi; k \in \mathbb{Z}.$
- $\alpha \in \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}; x \in \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\} : \operatorname{tg} x = \operatorname{tg} \alpha \Leftrightarrow x = \alpha + k'\pi, k' \in \mathbb{Z}$
- $\cos x = 0 \Leftrightarrow x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$
- $\sin x = 0 \Leftrightarrow x = k\pi, k \in \mathbb{Z}$
- $\cos x = 1 \Leftrightarrow x = 2k\pi, k \in \mathbb{Z}$